Repeat Paper Complete, Transparent & Interactive Systems for Highschool Mathematics

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Outline[1]

[1] Download PDF from `https://isac.miraheze.org/wiki/Publications_and_Theses`

Outline

A long-term project

- $\mathcal{ISAC}$-project[2] has been running for 3 decades, is now renamed to Isabelle/MAWEN, brings together 3 areas of expertise
  - Isabelle, the proof assistant, as software platform for ...
  - MAWEN, **Ma**thematical **W**ork **En**vironment for schools
  - 'accessibility' design for blind users
- Pedagogical concept is well-developed
- Software product is tried and tested in (public) schools
- Due to funding problems, the focus is on blind users now

[2] https://isac.miraheze.org/wiki/History

Outline

Proofs begin at school

- Learning at school ist different from academic learning
  - 14-year-olds need to start with concrete thinking tasks
    - numbers before variables
    - motivation by real world problems
    - *experience* with variables (+ bound in functions, . . .)
    - *experience* with formal rules
      (*experience* like in learning foreign languages)
  - Such *experiences* should prepare for abstraction:
    1. intuitive concept for variable, function, . . .
    2. intuitive concept for rule-application
       `explicit and formal matching ???`
    3. intuitive concepts for axiom, theorem, . . .
- Then students can be exposed to more abstraction at university
  - *Copying school lessons at university is boring!*
  - Mathematics lectures can once again follow the pattern: axiom / definition / theorem / proof

Outline

Now focus on blind students

- The new focus has been enforced by funding problems (discussed later)
- Blindness is a good prerequisite for . . .
  - for mathematical thinking
    - blind people access even web pages via structure
    - they are enforced to access mathematical formulas the same way: the usual Braille-display is only 40 characters
    - they develop *unusual ways of thinking*
  - for mathematical work
    - blind people's life-style excludes tennis, etc, . . .
    - their thinking is guided more by structure than by intuition
    - they can only work in a few jobs +? with mathematics?
- But mathematics software is not accessible . . .
  . . . despite MathJax, etc
  Even in specialised schoold gifted students drop out facing more complex formulas.

Outline

Transition from ISAC to MAWEN

- the educational design remains unchanged
- the Java front end is being phased out
  - because the formula representation was outdated.
  - Java front end was successful in teaching at schools,
  - but around 30 student projects are lost as a result
    `https://isac.miraheze.org/wiki/Credits`
- MAWEN is shifted under Isabelle's front-end
  - MAWEN is an Isabelle component now
  - and will therefore inherit the advances of Isabelle
- Isabelle/MAWEN has the following timetable
  - 2025/26: Authoring system for blind teachers
  - 2026/27: Starting point for serious trials in schools
  - 2027/28: Client-server solution (pupils have underpowered computers)

Outline

Interactive System I

The system presents maths as it appears in exercise books [3]

01 Problem (Biegelinie, [Biegelinien])
02 Specification:
03 Solution:
04 Problem (Biegelinie, [vonBelastungZu, Biegelinien])
05 [$V\,x = c + -1 \cdot q_0 \cdot x$,
06 $M_b\,x = c_2 + c \cdot x + \frac{-1 \cdot q_0}{2 \cdot x^2}$,
07 $\frac{d}{dx}\,y\,x = c_3 + \frac{-1}{EI} \cdot (c_2 \cdot x + \frac{c}{2 \cdot x^2} + \frac{-1 \cdot q_0}{6 \cdot x^3}$,
08 $y\,x = c_4 + c_3 \cdot x + \frac{-1}{EI} \cdot (\frac{c_2}{2} \cdot x^2 + \frac{c}{6} \cdot x^3 + \frac{-1 \cdot q_0}{24} \cdot x^4)$]
09 Problem (Biegelinie, [setzeRandbedingungen, Biegelinien])
10 [$L \cdot q_0 = c,\ 0 = \frac{2 \cdot c_2 + 2 \cdot L \cdot c + -1 L^2 \cdot q_0}{2},\ 0 = c_4,\ 0 = c_3$]
11 solveSystem ($L \cdot q_0 = c,\ 0 = \frac{2 \cdot c_2 + 2 \cdot L \cdot c + -1 \cdot L^2 \cdot q_0}{2},\ 0 = c_4,\ 0 = c_3$], $[c, c_2, c_3, c_4]$)
12 [$c = L \cdot q_0,\ c_2 = \frac{-1 \cdot L^2 \cdot q_0}{2},\ c_3 = 0,\ c_4 = 0$]
13 Take $y\,x = c_4 + c_3 \cdot x + \frac{-1}{EI} \cdot (\frac{c_2}{2} \cdot x^2 + \frac{c}{6} \cdot x^3 + \frac{-1 \cdot q_0}{24} \cdot x^4)$
14 $y\,x = c_4 + c_3 \cdot x + \frac{-1}{EI} \cdot (\frac{c_2}{2} \cdot x^2 + \frac{c}{6} \cdot x^3 + \frac{-1 \cdot q_0}{24} \cdot x^4)$
15 Substitute $[c, c_2, c_3, c_4]$
16 $y\,x = 0 + 0 \cdot x + \frac{-1}{EI} \cdot (\frac{\frac{-1 \cdot L^2 \cdot q_0}{2}}{2} \cdot x^2 + \frac{L \cdot q_0}{6} \cdot x^3 + \frac{-1 \cdot q_0}{24} \cdot x^4)$
17 Rewrite_Set_Inst ([(*bdv*, *x*)], *make_ratpoly_in*)
18 $y\,x = \frac{q_0 \cdot L^2}{4 \cdot EI} \cdot x^2 + \frac{L \cdot q_0}{6 \cdot EI} \cdot x^3 + \frac{q_0}{24 \cdot EI} \cdot x^4$
19 $y\,x = \frac{q_0 \cdot L^2}{4 \cdot EI} \cdot x^2 + \frac{L \cdot q_0}{6 \cdot EI} \cdot x^3 + \frac{q_0}{24 \cdot EI} \cdot x^4$

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[3] http://www.ist.tugraz.at/projects/isac/www/kbase/exp/exp_Statics_Biegel_Timischl.html

Interactive System II

The unique selling point of Isabelle/MAWEN: 'next step guidance'

- A practice problem is described by a formal specification (usually generated automatically by the system)
- The formal specification is linked to a solution method
- The method is defined by an Isabelle term as a program
- The programme is executed in a kind of debugger mode: the user is given control at every step of problem solving
- We call this procedure the 'Lucas Interpretation', after its originator [4]

This technique is welcome to be reused; it is described in detail in the paper.

[4] https://en.wikipedia.org/wiki/Peter_Lucas_(computer_scientist)

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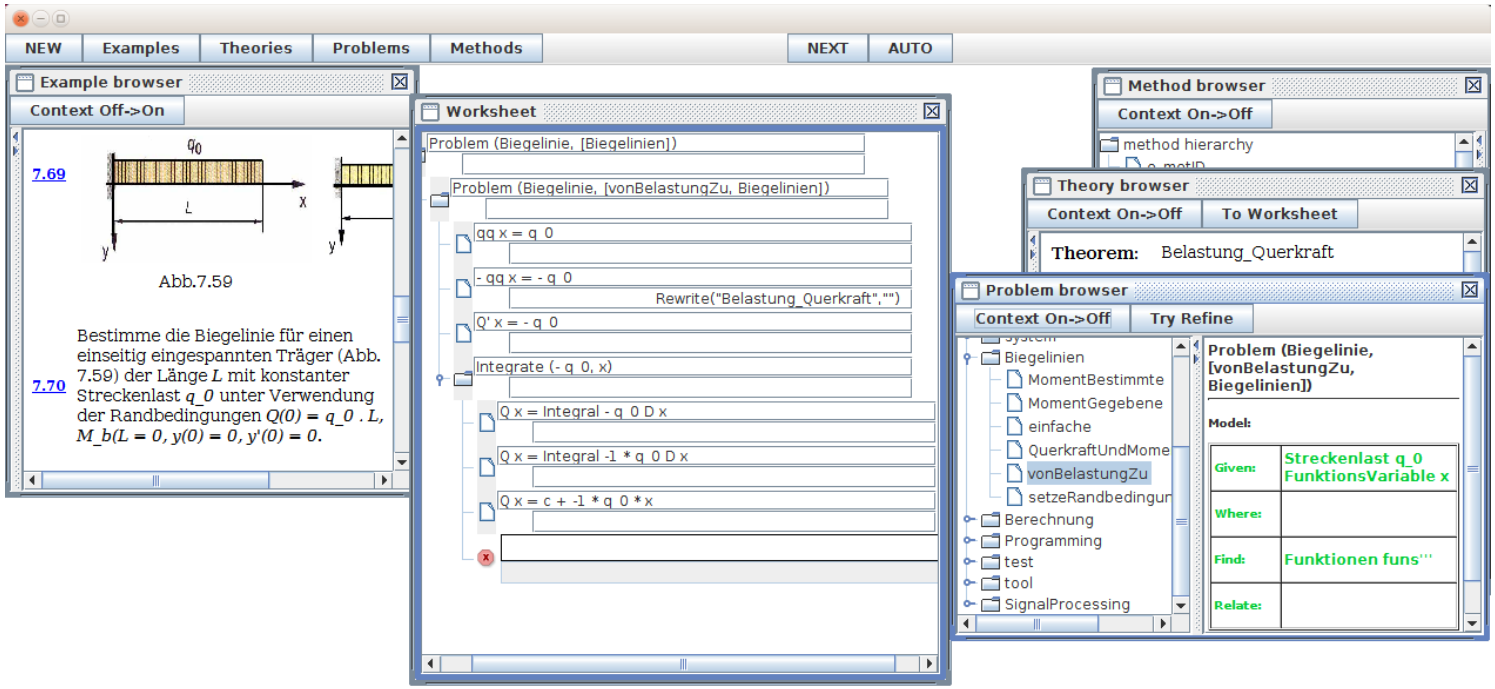
Transparent System I

The system reveals how it works in principle, the way the components work is transparent:



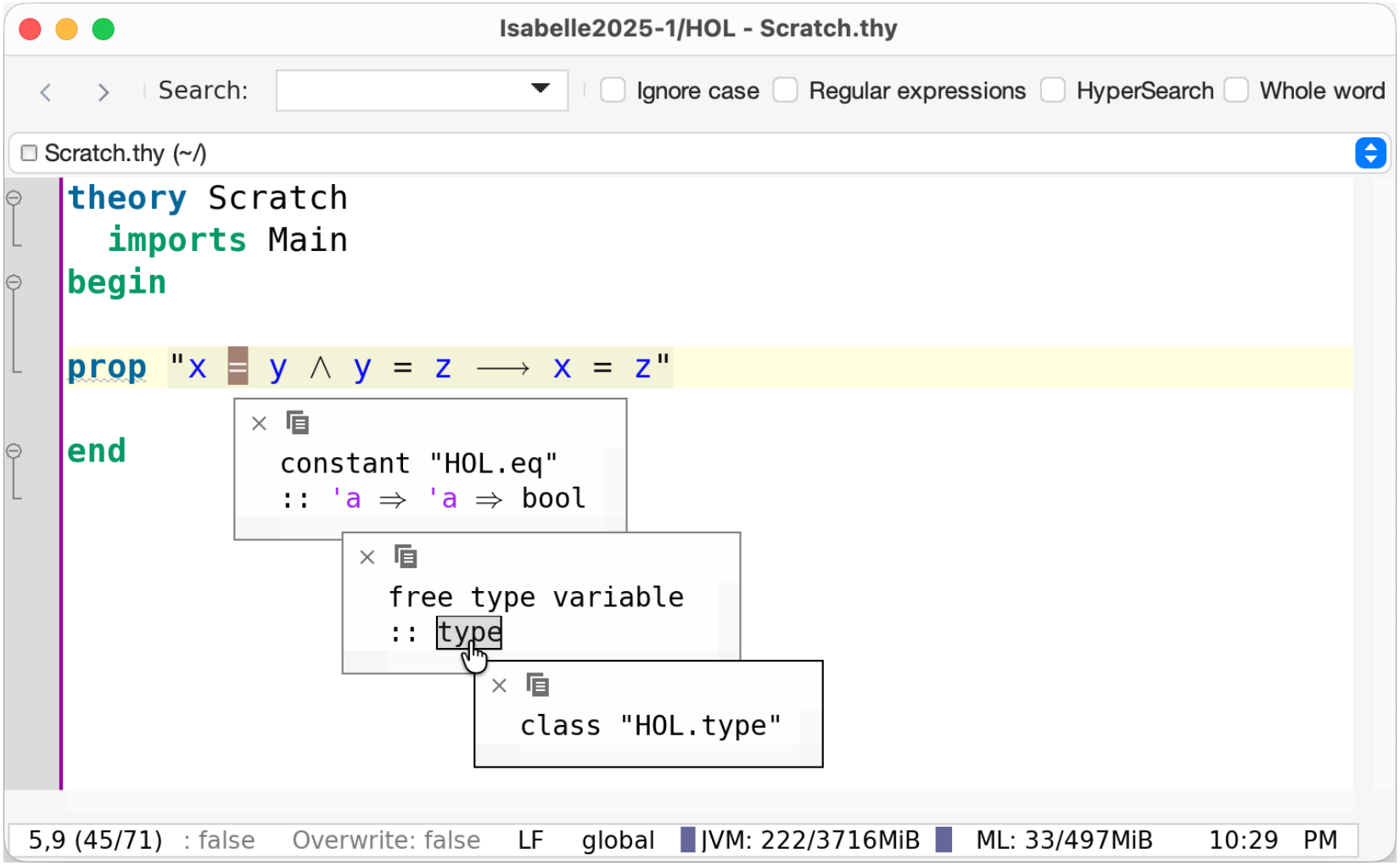
Transparent System II

Isabelle/MAWEN has its **own rewrite engine!**

- a *step* in a *calculation* consists of: $< t > \longrightarrow^{rule} < t' >$
- rules are collected in 'rule-sets'
  - e.g.
    $< 2 \cdot a + 3 \cdot b + 4 \cdot a > \longrightarrow^{ord-monom} < 2 \cdot a + 4 \cdot a + 3 \cdot b >$
    in order to add
    $< 2 \cdot a + 4 \cdot a + 3 \cdot b > \longrightarrow^{distr} < (2 + 4) \cdot a + 3 \cdot b >$
    requires douzens of steps
  - these are only *displayed on request*
- some steps *cannot be done by rewriting*
  - e.g canceling $< (1 + x)/(1 - x^2) > \longrightarrow < 1/(1 - x) >$
    is done by Euclid's Algorithm (implemented in ML)
  - the steps are shown by *reverse rewriting*

Transparent System III

Any mathematical object reveals the underlying formal knowledge (explanations to aid intuitive understanding are added later)



Outline

We need a Complete System

'Completeness' simply means, in this context, that

- ***one* software product** covers large parts of school maths *(many teachers are overwhelmed by having to use several software products)*
- The problems are **presented just as they are in a textbook** –
- *– with the difference that the system helps with solving them*
- Lucas-Interpretation requires **authoring**, with increasing levels of difficulty:
  1. Add your own practice examples
  2. Develop new problem types and methods
  3. Implement new theories, e.g. for 'beding lines'
- Ultimately, there will be challenges also in publishing.

Summary and Conclusion

- We have shown that
  - maths problem-solving can be modelled using software
  - proof assistants provide a suitable software basis
  - such software still requires a great deal of work
- In conclusion, we note that
  - the teaching of mathematics is currently taking a different direction in schools (the 'competence' approach, multiple-choice tests, etc.)
  - Isabelle/MAWEN can*not* be funded from public funds
  - the project therefore relies on private funding
- Nevertheless, we recommend that
  - other projects with similar objectives also might be launched
  - proof assistants also be used as the software basis

That might lead to a smoother transition from school maths to university, in mathematics and engineering sciences.

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Thank you for attention !

Surprises? Questions?